# **Lesson Objectives**

1. Basic Terms Involving Absolute Value
2. Solve an Absolute Value Equation
3. Solve an Absolute Value Inequality

# **Basic Terms** Involving Absolute Value

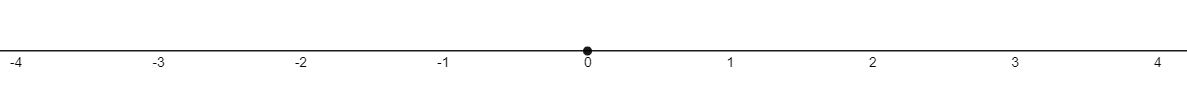
The **ABSOLUTE VALUE** of a number **IS** its **DISTANCE** from zero on a number line.

**ABSOLUTE VALUE IS DISTANCE**

What is always true about a DISTANCE? It is always POSITVE! It can NEVER be negative.

The same is true about the absolute value of a number. It is always positive.

Suppose you are standing at ZERO on a basic horizontal number line:



Where do you go so that you are a DISTANCE of 2 units away from zero? You go to either – 2 or 2.



That is to say, you can go either 2 units to the LEFT (– 2) or 2 units to the RIGHT (+2).

This situation can be modeled using an absolute value equation:

Where do you go… …the distance is 2 from zero

So with the equation: , you are to find numbers that are 2 units from zero.

The solutions can be modeled using two equations: *x* = – 2 or *x* = 2

Go 2 units to the LEFT of zero or go 2 units to the RIGHT

This type of solution, , can also be written in a simplified version: .

That format, , is read as “*x* equals plus or minus 2.”

So, remember the following: **ABSOLUTE VALUE IS DISTANCE**

* **EXAMPLE:** Solve the absolute value equation: [2.5.27]

Remember that absolute value is a distance. In this equation, the distance number is – 3.

But distance can’t be negative, so this equation has **NO SOLUTION.**

In general: means **“NO SOLUTION”**

# **Solve** an **Absolute Value Equation** (by hand – symbolically)

* **Step 1 – ISOLATE** the absolute value part, if needed.
* **Step 2 – INSPECT** the distance number (opposite the A.V. stuff).
  + If distance number is **NEGATIVE** – STOP! Equation has **NO SOLUTION**.
  + If distance number is **ZERO** – ignore A.V. bars; make **ONE** equation (**ONE** solution).
  + If distance number is **POSITIVE** – keep going; there will be **TWO** equations (**TWO** solutions).
* **Step 3 – “BRANCH OFF”** and make 2 separate equations

.

* **Step 4 – SOLVE** each equation.
* **Step 5 – WRITE** your solution(s).
* **EXAMPLE:** Solve the equation for *x*. [2.5.39]

**Step 1 – ISOLATE.** There is nothing attached to the outside of the A.V. part

(nothing multiplied; nothing added or subtracted)

**Step 2 – INSPECT.** Distance number, 0, means ignore A.V. bars & make 1 equation

**Step 3 – BRANCH. (**Not needed, since there is only one equation.)

**Step 4 – SOLVE.** converts to

so

**Step 5 – WRITE.** The solution is: .

* **EXAMPLE:** Solve for *b*. [2.5-8]

**Step 1 – ISOLATE.** Given: (Subtract 8)

(do NOT subtract 9…yet!)

**Step 2 – INSPECT.** The distance number, 2, is POSITIVE – make 2 equations

**Step 3 – BRANCH.** Make 2 separate equations:

Start BOTH equations with :

**Step 4 – SOLVE.** (Subtract 9)

**Step 5 – WRITE.**  **or**

* **EXAMPLE:** Find the solution set for the equation. [\*Blitzer 4.3.19]

**Step 1 – ISOLATE.** Given: (Subtract 5)

(Divide by 4)

(do NOT divide by 3….yet!)

**Step 2 – INSPECT.** The distance number, 8, is POSITIVE – make 2 equations

**Step 3 – BRANCH.** Make 2 separate equations:

Start BOTH equations with or

**Step 4 – SOLVE.** Solve each equation. (divide by 3)

**Step 5 – WRITE.** Write your solutions.  **or** also

# **Solve** an **Absolute Value Inequality** (by hand – symbolically)

## Solving a **Greater-Than** type **Absolute Value Inequality (More is “or”)**

* **EXAMPLE:** Solve the inequality. [2.5.83]

Focus for now on the information outside the inequality: **greater than 6**.

Where would you need to be so that you are **greater than 6** units away from zero?



Left piece is **less than – 6** or Right piece is **greater than 6**

In EACH piece graphed above, the distance is **greater than 6** units away from zero on the number line.

This is what we’ll use to set up the solution process for this inequality. **(More is “or”)**

( BOTH inequalities will start with )

(left piece) or (right piece)

(Solve each inequality)

(subtract 1)

or

(Divide by – 2)

(**REVERSE !!** – remember, you don’t always reverse)

or

(**VERY IMPORTANT!** Swap places to mimic number line; smaller on left, larger on right)

or

**WRITE** the solution. Set Builder Notation:

Interval Notation:

So, in general: If given setup is: or

Solution is: parentheses are used on 2 solutions

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If given setup is: or

Solution is: brackets are used on 2 solutions

## Solving a **Less-Than** type **Absolute Value Inequality (Less is “Nest”)**

* **EXAMPLE:** Solve the inequality. [2.5.75]

Focus for now on the information outside the inequality: **less than or equal to 8**.

Where would you need to be so that you are **less than or equal to 8** units away from zero?



This segment above shows staying a distance of **less than or equal to 8** units from zero.

So, notice that this graph shows all points with a distance **in between – 8** and **+8**.

That is written as a compound inequality:

This is what we’ll use to set up the solution process for this inequality.

Given: (**Less is “Nest”**)

Setup to Solve: “Nest” the 3 – 5x in the middle

Subtract 3:

Divide by – 5 and **REVERSE!**:

(Remember, you won’t always reverse)

You need to “pivot” (or “dab”) this inequality, to match the number line:

smaller ≤ *x* ≤ larger

**WRITE** the solution. Set Builder Notation:

Interval Notation:

So, in general: If given setup is:

Solution is: parentheses are used

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If given setup is:

Solution is: brackets are used

## Solve **Applications** with **Absolute Value Inequalities**

* **EXAMPLE:** The inequality describes the range of monthly average temperatures *T* in degrees Fahrenheit at a certain location. Find an equivalent expression and monthly average temperatures.

[2.5.117] (Less is “nest”)

Setup to solve:

Add 49:

The inequality is equivalent to: interval notation:

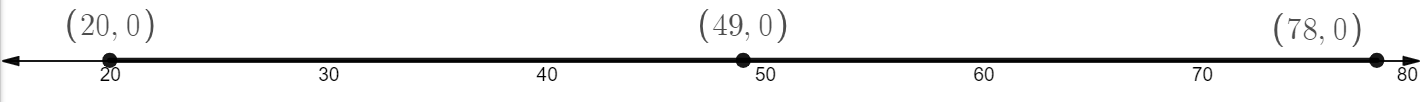
Interpret this solution: If the high and low monthly average temperatures satisfy the inequality, then the monthly averages are always within **29** degrees of **49**° F.

How do we get this? The “within” 29 degrees part represents a **DISTANCE**, so use the **DISTANCE** number (away from the inequality) for **“within”**.

The “of” 49° F part represents a **LOCATION** (center point), so use the value **inside** the absolute value.

“of” “within”

(center point, location) (distance)



Sources Used:

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